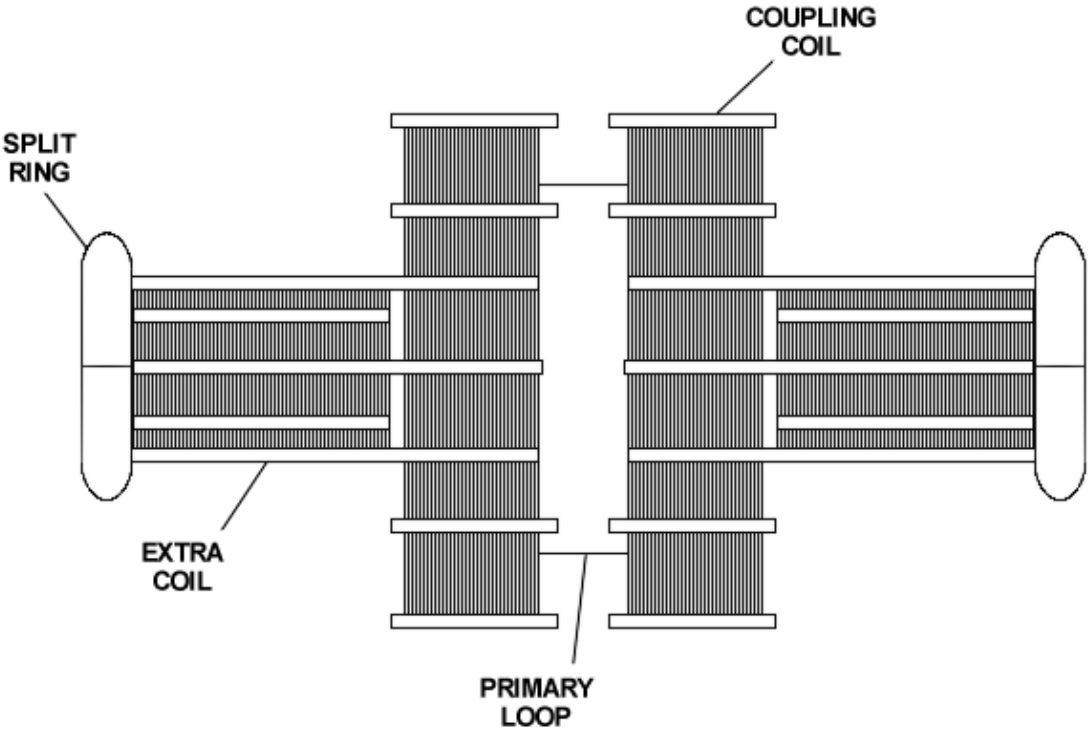


I) INTRODUCTION TO DIELECTRIC & MAGNETIC DISCHARGES IN ELECTRICAL WINDINGS
by Eric Dollard, ©1982

II) ELECTRICAL OSCILLATIONS IN ANTENNAE AND INDUCTION COILS
by John Miller, 1919



PART I

INTRODUCTION TO DIELECTRIC & MAGNETIC DISCHARGES IN ELECTRICAL WINDINGS

by Eric Dollard, ©1982

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4. THE LAWS OF LINES OF FORCE
5. FARADAY'S LINES OF FORCE THEORY
6. PHYSICAL CHARACTERISTICS OF LINES OF FORCE
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- 2) Table of Magnetic, Dielectric & Electronic Relations

PART II

ELECTRICAL OSCILLATIONS IN ANTENNAE & INDUCTION COILS

J.M. Miller

Proceedings, Institute of Radio Engineers. 1919

1) CAPACITANCE

The phenomena of capacitance is a type of electrical energy storage in the form of a field in an enclosed space. This space is typically bounded by two parallel metallic plates or two metallic foils on an intervening insulator or dielectric. A nearly infinite variety of more complex structures can exhibit capacity, as long as a difference in electric potential exists between various areas of the structure. The oscillating coil represents one possibility as to a capacitor of more complex form, and will be presented here.

2) CAPACITANCE INADEQUATELY EXPLAINED

The perception of capacitance as used today is wholly inadequate for the proper understanding of this effect. Steinmetz mentions this in his introductory book "Electric Discharges, Waves and impulses". To quote, "Unfortunately, to a large extent in dealing with dielectric fields the prehistoric conception of the electro-static charge (electron) on the conductor still exists, and by its use destroys the analogy between the two components of the electric field, the magnetic and the dielectric, and makes the consideration of dielectric fields unnecessarily complicated."

3) LINES OF FORCE AS REPRESENTATION OF DIELECTRICITY

Steinmetz continues, "There is obviously no more sense in thinking of the capacity current as current which charges the conductor with a quantity of electricity, than there is of speaking of the inductance voltage as charging the conductor with a quantity of magnetism. But the latter conception, together with the notion of a quantity of magnetism, etc., has vanished since Faraday's representation of the magnetic field by lines of force."

4) THE LAWS OF LINES OF FORCE

All the lines of magnetic force are closed upon themselves, all dielectric lines of force terminate on conductors, but may form closed loops in electromagnetic radiation.

These represent the basic laws of lines of force. It can be seen from these laws that any line of force cannot just end in Space.

5) FARADAY AND LINES OF FORCE THEORY

Faraday felt strongly that action at a distance is not possible thru empty space, or in other words, "matter cannot act where it is not." He considered space pervaded with lines of force. Almost everyone is familiar with the patterns formed by iron filings around a magnet. These filings act as numerous tiny compasses and orientate themselves along the lines of force existing around the poles of the magnet. Experiment has indicated that a magnetic field does possess a fibrous construct. By passing a coil of wire thru a strong magnetic field and listening to the coil output in headphones, the experimenter will notice a scraping noise. J. J. Thompson performed further experiments involving the ionization of gases that indicate the field is not continuous but fibrous (Electricity and Matter, 1906).

6) PHYSICAL CHARACTERISTICS OF LINES OF FORCE

Consider the space between poles of a magnet or capacitor as full of lines of electric force. See Fig. 1. These lines of force act as a quantity of stretched and mutually repellent springs. Anyone who has pushed together the like poles of two magnets has felt this springy mass. Observe Fig. 2. Notice the lines of force are more dense along AB in between poles, and that more lines on A are

facing B than are projecting outwards to infinity. Consider the effect of the lines of force on A. These lines are in a state of tension and pull on A. Because more are pulling on A towards B than those pulling on A away from B, we have the phenomena of physical attraction. Now observe Fig. 3. Notice now that the poles are like rather than unlike, more or all lines pull A away from B; the phenomena of physical repulsion.

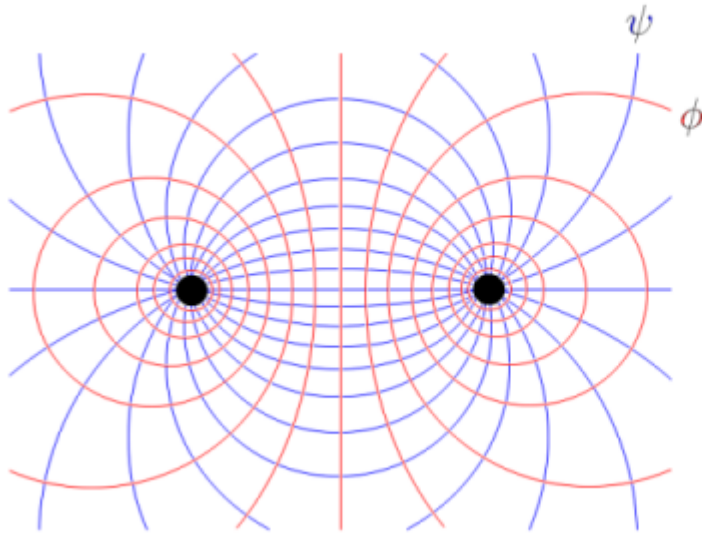


FIG. 1A - Electric Field of Circuit

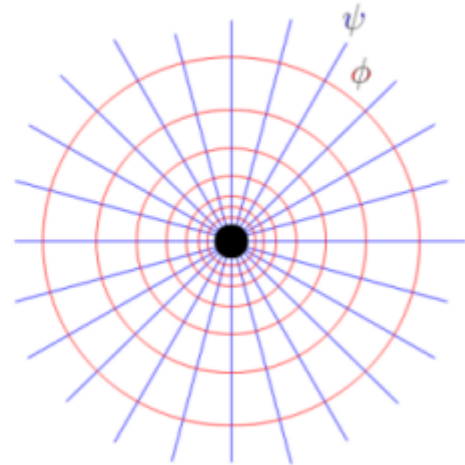


FIG. 1B - Electric Field of Conductor

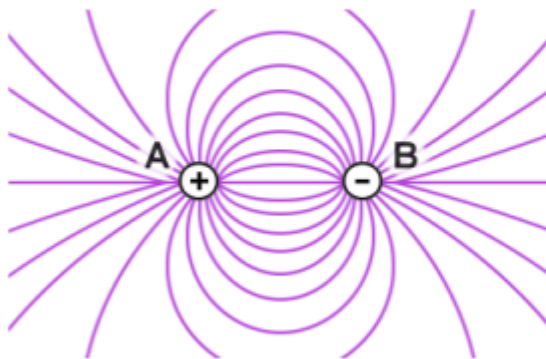


FIG. 2

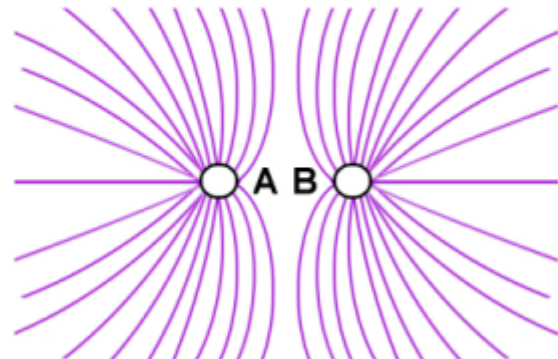


FIG. 3

7) MASS ASSOCIATED WITH LINES OF FORCE IN MOTION

The line of force can be more clearly understood by representing it as a tube of force or a long thin cylinder. Maxwell presented the idea that the tension of a tube of force is representative of electric force (volts/inch), and in addition to this tension, there is a medium through which these tubes pass. There exists a hydrostatic pressure against this media or ether. The value of this pressure is one half the product of dielectric and magnetic density. Then there is a pressure at right angles to an electric tube of force. If through the growth of a field the tubes of force spread sideways or in width, the broadside drag through the medium represents the magnetic reaction to growth in intensity of an electric current. However, if a tube of force is caused to move endwise, it will glide through the medium with little or no drag as little surface is offered. This possibly explains why no magnetic field is associated with certain experiments performed by Tesla involving the movement of energy with no accompanying magnetic field.

8) INDUCTANCE AS AN ANALOGY TO CAPACITY

Much of the mystery surrounding the workings of capacity can be cleared by close examination of inductance and how it can give rise to dielectric phenomena. Inductance represents energy storage in space as a magnetic field. The lines of force orientate themselves in closed loops surrounding the axis of current flow that has given rise to them. The larger the space between this current and its images or reflections, the more energy that can be stored in the resulting field.

9) MECHANISM OF STORING ENERGY MAGNETICALLY

The process of pushing these lines or loops outward, causing them to stretch, represents storing energy as in a rubber band. A given current strength will hold a loop of force at a given distance from conductor passing current hence no energy movement. If the flow of current increases, energy is absorbed by the field as the loops are then pushed outward at a corresponding velocity. Because energy is in motion an E.M.F. must accompany the current flow in order for it to represent power. The magnitude of this E.M.F. exactly corresponds to the velocity of the field. Then if the current ceases changing in magnitude thereby becoming constant, no E.M.F. accompanies it, as no power is being absorbed. However, if the current decreases it represents then a negative velocity of field as the loops contract. Because the E.M.F. corresponds exactly to velocity it reverses polarity and thereby reverses power so it now moves out of the field and into the current. Since no power is required to maintain a field, only current, the static or stationary field, represents stored energy.

10) THE LIMITS OF ZERO AND INFINITY

Many interesting features of inductance manifest themselves in the two limiting cases of trapping the energy or releasing it instantly. Since the power supply driving the current has resistance, when it is switched off the inductance drains its energy into this resistance that converts it into the form of heat. We will assume a perfect inductor that has no self resistance. If we remove the current supply by shorting the terminals of the inductor we have isolated it without interrupting any current. Since the collapse of field produces E.M.F. this E.M.F. will tend to manifest. However, a short circuit will not allow an E.M.F. to develop across it as it is zero resistance by definition. No E.M.F. can combine with current to form power, therefore, the energy will remain in the field. Any attempt to collapse forces increased current which pushes it right back out. This is one form of storage of energy.

11) INSTANT ENERGY RELEASE AS INFINITY

Very interesting (and dangerous) phenomena manifest themselves when the current path is interrupted, thereby causing infinite resistance to appear. In this case resistance is best represented by its inverse, conductance. The conductance is then zero. Because the current vanished instantly the field collapses at a velocity approaching that of light. As E.M.F. is directly released to velocity of flux, it tends towards infinity. Very powerful effects are produced because the field is attempting to maintain current by producing whatever E.M.F. required. If a considerable amount of energy exists, say several kilowatt hours* (250 KWH for lightning stroke), the ensuing discharge can produce most profound effects and can completely destroy inadequately protected apparatus.

* The energy utilized by an average household in the course of one day.

12) ANOTHER FORM OF ENERGY APPEARS

Through the rapid discharge of inductance a new force field appears that reduces the rate of inductive E.M.F. formation. This field is also represented by lines of force but these are of a different nature than those of magnetism. These lines of force are not a manifestation of current flow but of an electric compression or tension. This tension is termed voltage or potential difference.

13) DIELECTRIC ENERGY STORAGE SPATIALLY DIFFERENT THAN MAGNETIC ENERGY STORAGE

Unlike magnetism the energy is forced or compressed inwards rather than outwards. Dielectric lines of force push inward into internal space and along axis, rather than pushed outward broadside to axis as in the magnetic field. Because the lines are mutually repellent certain amounts of broadside or transverse motion can be expected but the phenomena is basically longitudinal. This gives rise to an interesting paradox that will be noticed with capacity. This is that the smaller the space bounded by the conducting structure the more energy that can be stored. This is the exact opposite of magnetism. With magnetism, the unit volumes of energy can be thought of as working in parallel but the unit volumes of energy in association with dielectricity can be thought of as working in series.

14) VOLTAGE IS TO DIELECTRICITY AS CURRENT IS TO MAGNETISM

With inductance the reaction to change of field is the production of voltage. The current is proportionate to the field strength only and not velocity of field. With capacity the field is produced not by current but voltage. This voltage must be accompanied by current in order for power to exist. The reaction of capacitance to change of applied force is the production of current. The current is directly proportional to the velocity of field strength. When voltage increases a reaction current flows into capacitance and thereby energy accumulates. If voltage does not change no current flows and the capacitance stores the energy which produced the field. If the voltage decreases then the reaction current reverses and energy flows out of the dielectric field.

As the voltage is withdrawn the compression within the bounded space is relieved. When the energy is fully dissipated the lines of force vanish.

15) AGAIN THE LIMITS ZERO AND INFINITY

Because the power supply which provides charging voltage has internal conductance, after it is switched off the current leaking through conductance drains the dielectric energy and converts it to heat. We will assume a perfect capacitance having no leak conductance. If we completely disconnect the voltage supply by open circuiting the terminals of the capacitor, no path for current flow exists by definition of an open circuit. If the field tends to expand it will tend towards the production of current. However, an open circuit will not allow the flow of current as it has zero conductance. Then any attempt towards field expansion raises the voltage which pushes the field back inwards. Therefore, energy will remain stored in the field. This energy can be drawn for use at any time. This is another form of energy storage.

16) INSTANT ENERGY RELEASE AT INFINITY

Phenomena of enormous magnitude manifest themselves when the criteria for voltage or potential difference is instantly disrupted, as with a short circuit. The effect is analogous with the open circuit of inductive current. Because the forcing voltage is instantly withdrawn the field

explodes against the bounding conductors with a velocity that may exceed light. Because the current is directly related to the velocity of field it jumps to infinity in its attempt to produce finite voltage across zero resistance. If considerable energy had resided in the dielectric force field, again let us say several K.W.H. the resulting explosion has almost inconceivable violence and can vaporize a conductor of substantial thickness instantly. Dielectric discharges of great speed and energy represent one of the most unpleasant experiences the electrical engineer encounters in practice.

17) ENERGY RETURNS TO MAGNETIC FORM

The powerful currents produced by the sudden expansion of a dielectric field naturally give rise to magnetic energy. The inertia of the magnetic field limits the rise of current to a realistic value. The capacitance dumps all its energy back into the magnetic field and the whole process starts over again. The inverse of the product of magnetic storage capacity and dielectric storage capacity represents the frequency or pitch at which this energy interchange occurs. This pitch may or may not contain overtones depending on the extent of conductors bounding the energies.

18) CHARACTERISTIC IMPEDANCE AS REPRESENTATION OF PULSATION OF ENERGY FIELD

The ratio of magnetic storage ability to that of the dielectric is called the characteristic impedance. This gives the ratio of maximum voltage to maximum current in the oscillatory structure. However, as the magnetic energy storage is outward and the dielectric storage is inward the total or double energy field pulsates in shape or size. The axis of this pulsation of force is the impedance of the system displaying oscillations and pulsation occurs at the frequency of oscillation.

19) ENERGY INTO MATTER

As the voltage or impedance is increased the emphasis is on the inward flux. If the impedance is high and rate of change is fast enough (perfect overtone series), it would seem possible the compression of the energy would transform it into matter and the reconversion of this matter into energy may or may not synchronize with the circle of oscillation. This is what may be considered supercapacitance, that is, stable longterm conversion into matter.

20) MISCONCEPTIONS OF PRESENT THEORY OF CAPACITANCE

The misconception that capacitance is the result of accumulating electrons has seriously distorted our view of dielectric phenomena. Also the theory of the velocity of light as a limit of energy flow, while adequate for magnetic force and material velocity, limits our ability to visualize or understand certain possibilities in electric phenomena. The true workings of free space capacitance can be best illustrated by the following example. It has been previously stated that dielectric lines of force must terminate on conductors. No line of force can end in space. If we take any conductor and remove it to the most remote portion of the universe, no lines of force can extend from this electrode to other conductors. It can have no free space capacity, regardless of the size of the electrode, therefore it can store no energy. This indicates that the free space capacitance of an object is the sum mutual capacity of it to all the conducting objects of the universe.

21) FREE SPACE INDUCTANCE IS INFINITE

Steinmetz in his book on the general or unified behavior of electricity "The Theory and Calculation of Transient Electric Phenomena and Oscillation," points out that the inductance of any unit length of an isolated filamentary conductor must be infinite. Because no image currents exist to

contain the magnetic field it can grow to infinite size. This large quantity of energy cannot be quickly retrieved due to the finite velocity of propagation of the magnetic field. This gives a non reactive or energy component to the inductance which is called electromagnetic radiation.

22) WORK OF TESLA, STEINMETZ AND FARADAY

In the aforementioned books of Steinmetz he develops some rather unique equations for capacity. Tesla devoted an enormous portion of his efforts to dielectric phenomena and made numerous remarkable discovers in this area. Much of this work is yet to be fully uncovered. It is my contention that the phenomena of dielectricity is wide open for profound discovery. It is ironic that we have abandoned the lines of force concept associated with a phenomena measured in the units called farads after Farady, whose insight into forces and fields has led to the possibility of visualization of the electrical phenomena.

23) QUESTION AS TO THE VELOCITY OF DIELECTRIC FLUX

It has been stated that all magnetic lines of force must be closed upon themselves, and that all dielectric lines of force must terminate upon a conducting surface. It can be infered from these two basic laws that no line of force can terminate in free space. This creates an interesting question as to the state of dielectric flux lines before the field has had time to propagate to the neutral conductor. During this time it would seem that the lines of force, not having reached the distant neutral conductor would end in space at their advancing wave front. It could be concluded that either the lines of force propagate instantly or always exist and are modified by the electric force, or voltage. It is possible that additional or conjugate space exists within the same boundaries as ordinary space. The properties of lines of force within this conjugate space may not obey the laws of normally conceived space.

IMPORTANT REFERENCE MATERIAL

1. "Electricity and Matter," J. J. Thompson
New York, 1906, Scribner's Sons, and 1904, Yale University
2. "Elementary Lectures on Electric Discharges, Waves, and Impulses and other Transients."
C. P. Steinmetz, second edition, 1914, McGraw-Hill
3. "Theory and Calculation of Transient Electric Phenomena and Oscillations," C. P.
Steinmetz, third edition, 1920, McGraw-Hill. Section III Transients in Space, Chapter VIII,
Velocity of Propagation of Electric Field

Table I

Magnetic Field	Dielectric Field
<p>Magnetic flux: $\varphi = Li \ 10^8$ lines of magnetic force.</p>	<p>Dielectric flux: $\psi = Ce$ lines of dielectric force.</p>
<p>Inductance voltage: $e' = n \frac{d\phi}{dt} 10^{-8} = L \frac{di}{dt}$ volts.</p>	<p>Capacity current: $i' = n \frac{d\psi}{dt} = C \frac{di}{dt}$ amperes.</p>
<p>Magnetic energy: $w = \frac{Li^2}{2}$ joules.</p>	<p>Dielectric energy: $w = \frac{Ce^2}{2}$ joules.</p>
<p>Magnetomotive force: $F = ni$ ampere turns.</p>	<p>Electromotive force: $e =$ volts.</p>
<p>Magnetizing force: $f = \frac{F}{l}$ ampere turns per cm.</p>	<p>Electrifying force or voltage gradient: $G = \frac{e}{l}$ volts per cm.</p>
<p>Magnetic-field intensity: $\mathcal{H} = 4\pi f \ 10^{-1}$ lines of magnetic force per cm².</p>	<p>Dielectric-field intensity: $K = \frac{G}{4\pi v^2}$ lines of dielectric force per cm².</p>
<p>Magnetic density: $\mathcal{B} = \mu\mathcal{H}$ lines of magnetic force per cm².</p>	<p>Dielectric density: $D = kK$ lines of dielectric force per cm².</p>
<p>Permeability: μ</p>	<p>Permittivity or specific capacity: κ</p>
<p>Magnetic flux: $\varphi = AB$ lines of magnetic force.</p>	<p>Dielectric flux: $\psi = AD$ lines of dielectric force.</p>
$v = 3 \times 10^{10} =$ velocity of light	

Table II

Magnetic Circuit	Dielectric Circuit	Electric Circuit
Magnetic Flux (magnetic current): ϕ = lines of magnetic force.	Dielectric flux (dielectric current): ψ = lines of dielectric force.	Electric current: i = electric current.
Magnetomotive force: $F = ni$ ampere turns.	Electromotive force: e = volts.	Voltage: e = volts.
Permeance: $M = \frac{\phi}{4\pi F}$	Permittance or capacity: $C = \frac{4\pi v^2 \psi}{e}$ farads.	Conductance: $g = \frac{i}{e}$ mhos.
Inductance: $L = \frac{n^2 \phi}{F} 10^{-8} = \frac{n \phi}{i} 10^{-8}$ henry.	(Elastance ?): $\frac{1}{C} = \frac{e}{4\pi v^2 \psi}$	Resistance: $r = \frac{e}{i}$ ohms.
Reluctance: $R = \frac{F}{\phi}$	Dielectric energy: $w = \frac{Ce^2}{2} = \frac{e\psi}{2}$ joules.	Electric power: $p = ri^2 = ge^2 = ei$ watts.
Magnetic energy: $w = \frac{Li^2}{2} = \frac{F\phi}{2} 10^{-8}$ joules.	Dielectric density: $D = \frac{\psi}{A} = \kappa K$ lines per cm ² .	Electric-current density: $I = \frac{i}{A} = \gamma G$ amperes per cm ² .
Magnetic density: $\mathcal{B} = \frac{\phi}{A} = \mu \mathcal{H}$ lines per cm ² .	Dielectric gradient: $G = \frac{e}{l}$ volts per cm.	Electric gradient: $G = \frac{e}{l}$ volts per cm.
Magnetizing force: $f = \frac{F}{l}$ ampere turns per cm.	Dielectric field-intensity: $K = \frac{G}{4\pi v^2}$	Conductivity: $\gamma = \frac{I}{G}$ mhos-cm.
Magnetic-field intensity: $\mathcal{H} = .4\pi f$	Permittivity or specific capacity: $\kappa = \frac{D}{K}$	Resistivity: $\rho = \frac{1}{\gamma} = \frac{G}{I}$ ohms-cm.
Permeability: $\mu = \mathcal{B} / \mathcal{H}$	(Elasticity ?): $\frac{1}{\kappa} = \frac{K}{D}$	Specific power: $p_0 = \rho I^2 = G^2 = GI$ watts per cm ³ .
Reluctivity: $\rho = f / \mathcal{B}$	Specific dielectric energy: $w_0 = \frac{\kappa G^2}{4\pi v^2} = \frac{GD}{2}$ joules per cm ³ .	
Specific magnetic energy: $w_0 = \frac{.4\pi \mu f^2}{2} = f\mathcal{B} / 2 10^{-8}$ joules per cm ³ .		

Table of Units, Symbols, and Dimensions

	Quantity	Symbol	mks Unit Rationalized	Defining Equation	Dimensional Formula Exponents of				cgs emu	No. of emu No. of mks	cgs esu	No. of esu No. of mks	No. of esu No. of emu
					L	M	T	Q					
1	Length	L	m		1	0	0	0	cm	10^2	cm	10^2	1
2	Area	A	m ²	$A = L^2$	2	0	0	0	cm ²	10^4	cm ²	10^4	1
3	Volume	v	m ³	$v = L^3$	3	0	0	0	cm ³	10^6	cm ³	10^6	1
4	Mass	M, m	kilogram		0	1	0	0	gram	10^3	gram	10^3	1
5	Time	T, t	second		0	0	1	0	second	1	second	1	1
6	Velocity	v	m / sec	$v = L / T$	1	0	-1	0	cm / sec	10^2	cm / sec	10^6	1
7	Acceleration	a	m / sec ²	$a = L / T^2$	1	0	-2	0	cm / sec ²	10^2	cm / sec ²	10^2	1
8	Force	F	newton	$F = Ma$	1	1	-2	0	dyne	10^5	dyne	10^5	1
9	Energy	W	joule	$W = FL$	2	1	-2	0	erg	10^7	erg	10^7	1
10	Power	P	watt	$P = W / T$	2	1	-3	0	erg / sec	10^7	erg / sec	10^7	1
11	Charge	Q, q	coulomb	$F = Q^2 / (4\pi\epsilon_0 L^2)$	0	0	0	1	abcoulomb	10^{-1}	statcoulomb	$10c$	$100c$
12	Dielectric constant of free space	ϵ_0	farad / m	$\epsilon_0 = 1 / (\mu_0 c^2)$	-3	-1	2	2			1	$4\pi c^2 / 10^7$	
13	Dielectric constant	ϵ	farad / m		-3	-1	2	2					
14	relative	ϵ_r	numeric	$\epsilon_r = \epsilon / \epsilon_0$	0	0	0	0				1	
	Charge density												
15	volume	ρ	coulomb / m ³	$\rho = Q / v$	-3	0	0	1	abcoulomb / cm ³	10^{-7}	statcoulomb / cm ³	$c / 10^5$	$100c$
16	surface	ρ_s	coulomb / m ²	$\rho_s = Q / A$	-2	0	0	1	abcoulomb / cm ²	10^{-5}	statcoulomb / cm ²	$c / 10^3$	$100c$
17	line	ρ_l	coulomb / m	$\rho_l = Q / L$	-1	0	0	1	abcoulomb / cm	10^{-3}	statcoulomb / cm	$c / 10$	$100c$
18	Electric intensity	E	volt / m	$E = F / Q = -V / L$	1	1	-2	-1	abvolt / cm	10^6	statvolt / cm	$10^4 / c$	$1 / (100c)$
19	Electric flux density	D	coulomb / m ²	$D = \epsilon E = \psi / A$	-2	0	0	1		$4\pi / 10^5$		$4\pi c / 10^3$	$100c$
20	Electric flux	ψ	coulomb	$\psi = DA$	0	0	0	1		$4\pi / 10$		$4\pi 10c$	$100c$
21	Electric potential	V	volt	$V = -EL$	2	1	-2	-1	abvolt	10^6	statvolt	$10^6 / c$	$1 / (100c)$
22	EMF	V_g	volt	$V_g = -d\phi / dt$	2	1	-2	-1	abvolt	10^8	statvolt	$10^6 / c$	$1 / (100c)$
23	Capacitance	C	farad	$C = Q / V$	-2	-1	2	2	abfarad	10^{-9}	statfarad	$c^2 / 10^5$	$(100c)^2$
24	Current	I, i	ampere	$I = Q / T$	0	0	-1	1	abampere	10^{-1}	statampere	$10c$	$100c$
25	Current density	J	ampere / m ²	$J = I / A$	-2	0	-1	1	abampere / cm ²	10^{-5}	statampere / cm ²	$c / 10^3$	$100c$
26	Resistance	R	ohm	$R = V / I$	2	1	-1	-2	abohm	10^9	statohm	$10^5 / c^2$	$1 / (100c)^2$
27	Resistivity	ρ	ohm-m	$\rho = RA / L$	3	1	-1	-2	abohm-cm	10^{11}	statohm-cm	$10^7 / c^2$	$1 / (100c)^2$
28	Conductance	G	mho	$G = 1 / R$	-2	-1	1	2	abmho	10^{-9}	statmho	$c^2 / 10^5$	$(100c)^2$
29	Conductivity	σ	mho / m	$\sigma = 1 / \rho = J / E$	-3	-1	1	2	abmho / cm	10^{-11}	statmho / cm	$c^2 / 10^7$	$(100c)^2$
30	Electric polarization	P	coulomb / m ²	$P = D - \epsilon_0 E = \rho L$	-2	0	0	1	abcoulomb / cm ²	10^{-5}	statcoulomb / cm ²	$c / 10^3$	$100c$
31	Electric susceptibility	x_e	farad / m	$x_e = P / E = \epsilon_0 (\epsilon_r - 1)$	-3	-1	2	2			1	$4\pi c^2 / 10^7$	
32	Electric dipole moment	m_e	coulomb-m	$m_e = QL$	1	0	0	1			statcoulomb-cm	$10^3 c$	
33	Electric energy density	ω_e	joule / m ³	$\omega_e = DE / 2$	-1	1	-2	0	erg / cm ³	1	erg / cm ³	10	1

	Quantity	Symbol	mks Unit Rationalized	Defining Equation	Dimensional Formula Exponents of				cgs emu	No. of emu No. of mks	cgs esu	No. of esu No. of mks	No. of esu No. of emu
					L	M	T	Q					
34	Permeability of free space	μ_0	henry / m	$\mu_0 = 4\pi / 10^7$	1	1	0	-2		$10^7 / 4\pi$			
35	Permeability	μ	henry / m	$\mu = B / H$	1	1	0	-2					
36	relative	μ_r	numeric	$\mu_r = \mu / \mu_0$	0	0	0	0		1			
37	Magnetic pole	p	weber	$p = A (B - B_0)$	2	1	-1	-1	pole = maxwell / 4π	$10^3 / 4\pi$			
38	Magnetic moment	m	weber-m	$m = pL$	3	1	-1	-1	pole-cm	$10^{10} / 4\pi$			
39	Magnetic intensity	H	ampere / m or newton / weber	$H = U / L \text{ or } F / p$	-1	0	-1	1	oersted or gilbert / cm	$4\pi / 10^3$			
40	Magnetic flux density	B	weber / m ²	$B = \mu H = \phi / A$	0	1	-1	-1	gauss or maxwell / cm ²	10^4			
41	Magnetic flux	ϕ	weber	$\phi = BA = V_g T$	2	1	-1	-1	maxwell	10^8			
42	Magnetic potential	U	ampere	$U = \mathcal{F} = HL$	0	0	-1	1	gilbert	$4\pi / 10$			
43	MMF	\mathcal{F}	ampere	$\mathcal{F} = I$	0	0	-1	1	gilbert	$4\pi / 10$			
44	Intensity of magnetization	M	weber / m ²	$M = B - B_0 = m / L^3$	0	1	-1	-1	pole / cm ² or gauss / 4π	$104 / 4\pi$			
Inductance													
45	self	L	henry	$L = \phi / I$	2	1	0	-2	abhenry	10^9		$10^5 / c^2$	$1 / (100c)^2$
46	mutual	M	henry	$M = \phi / I = W / I^2$	2	1	0	-2	abhenry	10^9		$10^5 / c^2$	$1 / (100c)^2$
47	Reluctance	\mathcal{R}	ampere / weber	$\mathcal{R} = \mathcal{F} / \phi$	-2	-1	0	2					
48	Reluctivity	ν	meter / henry	$\nu = 1 / \mu$	-1	-1	0	2					
49	Permeance	\mathcal{P}	weber / amp	$\mathcal{P} = 1 / \mathcal{R}$	2	1	0	-2					
50	Permittivity	μ	henry / meter	$\mu = 1 / \nu$	1	1	0	-2					
51	EMF	V_g	volt	$V_g = -d\phi / dt$	2	1	-2	-1	abvolt	10^8	statvolt	$10^8 / c$	$1 / (100c)$
52	Poynting's vector	\mathcal{P}	watts / m ²	$\mathcal{P} = EH$	0	1	-3	0	abwatt / cm ²	10^3	statwatt / cm ²	10^3	1
53	Magnetic energy density	ω_m	joule / m ³	$\omega_m = HB / 2$	-1	1	-2	0	erg / cm ³	10	erg / cm ³	10	1
54	Magnetic susceptibility	x_m	henry / m	$x_m = M / H$ $= \mu_0 (\mu_r - 1)$	1	1	0	-2	henry / m	$10^7 / 4\pi$			

$\mu_0 = 4\pi / 10^7$ henrys / m. For $c = 2.998 \times 10^8$ meters / sec, $\epsilon_0 = 1 / \mu_0 c^2 = 10^7 / (4\pi c^2) = 8.854 \times 10^{-12}$ farad / meter

For $c \sim 3 \times 10^8$ meters / sec, $\epsilon_0 \sim 1 / (36\pi 10^9)$ farad / meter

$c^2 = 8.988 \times 10^{16} \sim 9 \times 10^{16}$

ELECTRICAL OSCILLATIONS IN ANTENNAS AND INDUCTANCE COILS

By John M. Miller

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I. INTRODUCTION

A modern radiotelegraphic antenna generally consists of two portions, a vertical portion or "lead-in" and a horizontal portion or "aerial." At the lower end of the lead-in, coils or condensers or both are inserted to modify the natural frequency of the electrical oscillations in the system. When oscillating, the current throughout the entire lead-in is nearly constant and the inductances, capacities, and resistances in this portion may be considered as localized or lumped. In the horizontal portion, however, both the strength of the current and the voltage to earth vary from point to point and the distribution of current and voltage varies with the frequency. The inductance, capacity, and resistance of this portion must therefore be considered as distributed throughout its extent and its effective inductance, capacity, and resistance will depend upon the frequency. On this account the mathematical treatment of the oscillations of an antenna is not as simple as that which applies to ordinary circuits in which all of the inductances and capacities may be considered as lumped.

The theory of circuits having uniformly distributed electrical characteristics such as cables, telephone lines, and transmission lines has been applied to antennas. The results of this theory do not seem to have been clearly brought out; hazy and sometimes erroneous ideas appear to be current in the literature, textbooks, and in the radio world in general so that the methods of antenna measurements are on a dubious footing. It is hoped that this paper may clear up some of these points. No attempt has been made to show how accurately this theory applies to actual antennas.

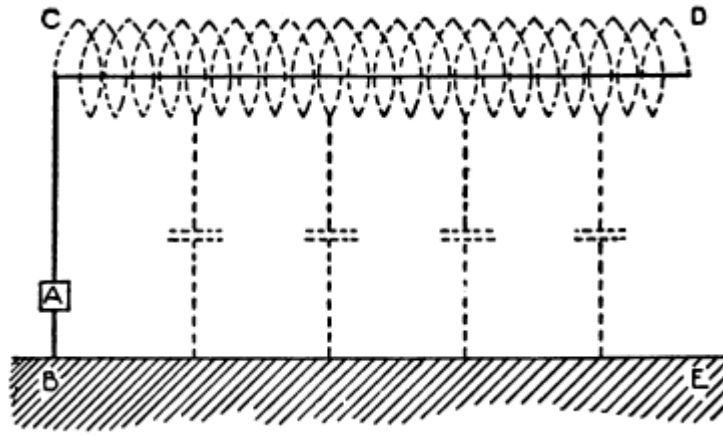


FIG 1. - Antenna represented as a line with uniform distribution of inductance and capacity

The aerial-ground portion of the antenna, or aerial for short (CD in Fig. 1), will be treated as a line with uniformly distributed inductance, capacity, and resistance. As is common in the treatment of radio circuits the resistance will be considered to be so low as not to affect the frequency of the oscillations or the distribution of current and voltage. The lead-in, BC in Fig. 1, will be considered to be free from inductance or capacity excepting as inductance coils or condensers are inserted at A to modify the oscillations.

An inductance coil, particularly if a long single-layer solenoid, may also be treated from the standpoint of the transmission-line theory. The theoretical results obtained furnish an interesting explanation of certain well-known experimental results.

II. CIRCUIT WITH UNIFORMLY DISTRIBUTED INDUCTANCE AND CAPACITY

The theory, generally applicable to all circuits with uniformly distributed inductance and capacity, will be developed for the case of two parallel wires. The wires (Fig. 2) are of length l and of low resistance. The inductance per unit length L_1 , is defined by the flux of magnetic force between the wires per unit of length that there would be if a steady current of 1 ampere were flowing in opposite directions in the two wires. The capacity per unit length C_1 is defined by the charge that there would be on a unit length of one of the wires if a constant emf of 1 volt were impressed between the wires. Further, the quantity $L_0 = l L_1$ would be the total inductance of the circuit if the current flow were the same at all parts. This would be the case if a constant or slowly

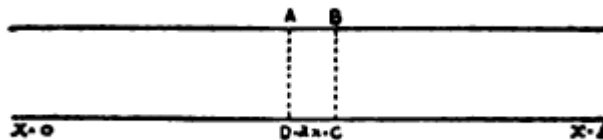


FIG. 2

alternating voltage were applied at $x = 0$ and the far end ($x = l$) short-circuited. The quantity $C_0 = l C_1$ would represent the total capacity between the wires if a constant or slowly alternating voltage were applied at $x = 0$ and the far end were open.

Let it be assumed, without defining the condition of the circuit at $x = l$, that a sinusoidal emf of periodicity $\omega = 2\pi f$ is impressed at $x = 0$ giving rise to a current of instantaneous value i at A and

a voltage between A and D equal to v . At B the current will be $i + \frac{\delta i}{\delta x} dx$ and the voltage from B to C will be $v + \frac{\delta v}{\delta x} dx$.

The voltage around the rectangle $ABCD$ will be equal to the rate of decrease of the induction through the rectangle, hence

$$\left(v + \frac{\delta v}{\delta x} dx\right) - v = -\frac{\delta}{\delta t} (L_1 i dx)$$

$$\frac{\delta v}{\delta x} = -L_1 \frac{\delta i}{\delta t} \quad (1)$$

Further, the rate of increase of the charge q on the elementary length of wire AB will be equal to the excess in the current flowing in at A over that flowing out at B .

Hence

$$\frac{\delta q}{\delta t} = \frac{d}{dt} (C_1 v dx) = i - \left(i + \frac{\delta i}{\delta x} dx\right)$$

$$-\frac{\delta i}{\delta x} = C_1 \frac{\delta v}{\delta t} \quad (2)$$

These equations (1) and (2) determine the propagation of the current and voltage waves along the wires. In the case of sinusoidal waves, the expressions

$$v = \cos \omega t (A \cos \omega \sqrt{C_1 L_1} x + B \sin \omega \sqrt{C_1 L_1} x) \quad (3)$$

$$i = \sin \omega t \sqrt{\frac{C_1}{L_1}} (A \sin \omega \sqrt{C_1 L_1} x - B \cos \omega \sqrt{C_1 L_1} x) \quad (4)$$

are solutions of the above equations as may be verified by substitution. The quantities A and B are constants depending upon the terminal conditions. The velocity of propagation of the waves, at high frequencies is $V = \frac{1}{\sqrt{C_1 L_1}}$.

III. THE ANTENNA

1. REACTANCE OF THE AERIAL-GROUND PORTION

Applying equations (3) and (4) to the aerial of an antenna and assuming that $x = 0$ is the lead-in end while $x = l$ is the far end which is open, we may introduce the condition that the current is zero for $x = l$. From (4)

$$\frac{A}{B} = \cot \omega \sqrt{C_1 L_1} l \quad (5)$$

Now the reactance of the aerial, which includes all of the antenna but the lead-in, is given by the current and voltage at $x = 0$. These are, from (3), (4), and (5).

$$V_0 = A \cos \omega t = B \cot \omega \sqrt{C_1 L_1} l \cos \omega t$$

$$i_0 = -\sqrt{\frac{C_1}{L_1}} B \sin \omega t$$

The current leads the voltage when the cotangent is positive and lags when the cotangent is negative. The reactance of the aerial, given by the ratio of the maximum values of v_0 to i_0 , is

$$X = -\sqrt{\frac{L_1}{C_1}} \cot \omega \sqrt{C_1 L_1} l$$

or in terms of $C_0 = lC_1$ and $L_0 = lL_1$

$$X = -\sqrt{\frac{L_0}{C_0}} \cot \omega \sqrt{C_0 L_0} \quad (6)$$

or since

$$V = \frac{1}{\sqrt{L_1 C_1}}$$

$$X = -L_1 V \cot \omega \sqrt{C_1 L_1} l \quad \text{as given by J. S. Stone.}^1$$

At low frequencies the reactance is negative and hence the aerial behaves as a capacity. At the frequency $f = \frac{1}{4\sqrt{C_0 L_0}}$

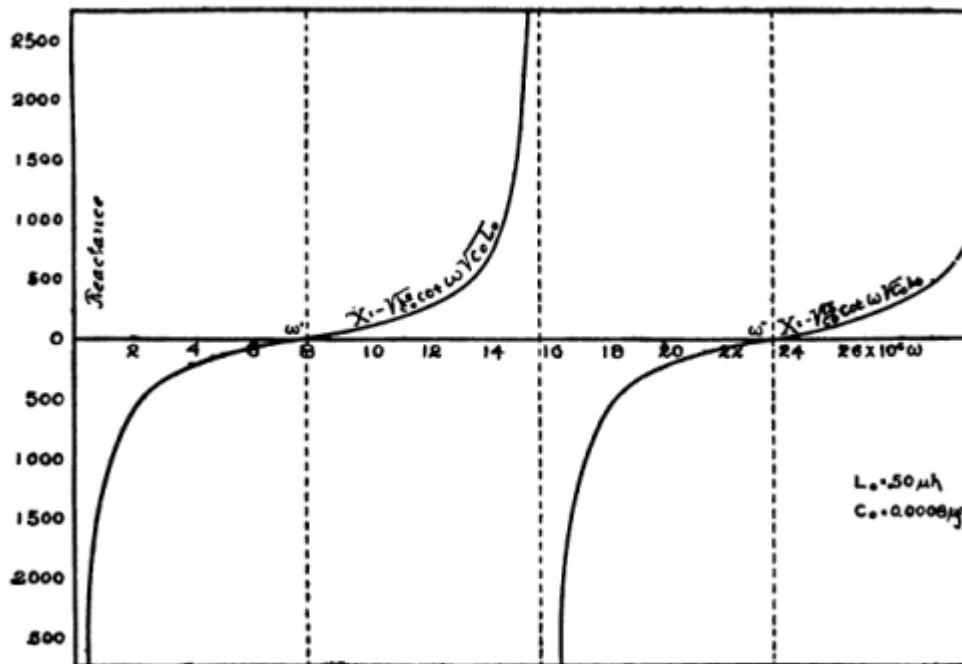


FIG. 3 - Variation of the reactance of the aerial of an antenna with the frequency

¹ J. S. Stone, Trans. Int. Congress, St. Louis, 8, p. 555; 1904.

the reactance becomes zero and beyond this frequency is positive or inductive up to the frequency $f = \frac{1}{2\sqrt{C_0 L_0}}$ at which the reactance becomes infinite. This variation of the aerial reactance with the frequency is shown by the cotangent curves in Fig. 3.

2. NATURAL FREQUENCIES OF OSCILLATION

Those frequencies at which the reactance of the aerial, as given by equation (6), becomes equal to zero are the natural frequencies of oscillation of the antenna (or frequencies of resonance) when the lead-in is of zero reactance. They are given in Fig. 3 by the points of intersection of the cotangent curves with the axis of ordinates and by the equation

$$f = \frac{m}{4\sqrt{C_0 L_0}}; m=1, 3, 5, \text{etc.}$$

The corresponding wave lengths are given by

$$\lambda = \frac{V}{f} = \frac{l}{f\sqrt{C_0 L_0}} = \frac{4l}{m}$$

that is, $4/1, 4/3, 4/5, 4/7, \text{etc.}$, times the length of the aerial. If, however, the lead-in has a reactance X_X , the natural frequencies

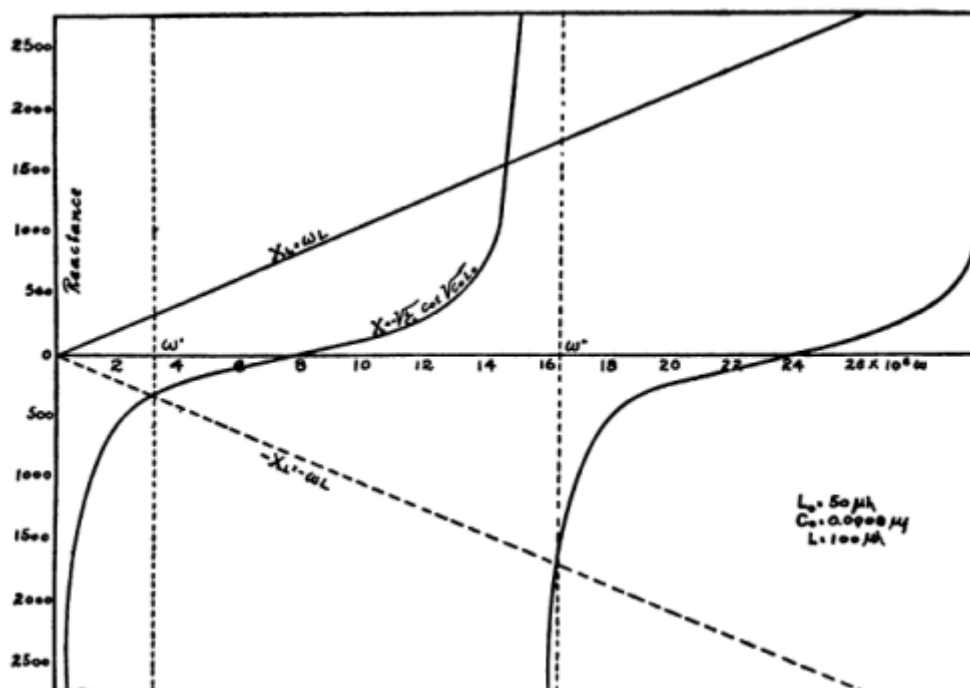


FIG. 4 - Curves of aerial and loading coil reactance

of oscillation are determined by the condition that the total reactance of lead-in plus aerial shall be zero; that is,

$$X_X + X = 0$$

provided that the reactances are in series with the driving emf.

(a) *Loading Coil in Lead-in.* - The most important practical case is that in which an inductance coil is inserted in the lead—in. If the coil has an inductance L , its reactance $X_L + \omega L$. This is a positive reactance increasing linearly with the frequency and represented in Fig. 4 by a solid line. Those frequencies at which the reactance of the coil is equal numerically but opposite in sign to the reactance of the aerial, are the natural frequencies of oscillation of the loaded antenna since the total reactance $X_L + X = 0$. Graphically, these frequencies are determined by the intersection of the straight line - $X_L = \omega L$ (shown by a dash line in Fig. 4) with the cotangent curves representing X . It is evident that the frequency is lowered by the insertion of the loading coil and that the higher natural frequencies of oscillation are no longer integral multiples of the lowest frequency.

The condition $X_L + X = 0$, which determines the natural frequencies of oscillation, leads to the equation

$$\omega L - \sqrt{\frac{L_0}{C_0}} \cot \omega \sqrt{C_0} L_0 = 0$$

TABLE 1 – Data for loaded antenna calculations

$\frac{L}{L_0}$	$\omega \sqrt{C_0} L_0$	$\frac{1}{\sqrt{\frac{L}{L_0} + \frac{1}{3}}}$	Difference, per cent	$\frac{L}{L_0}$	$\omega \sqrt{C_0} L_0$	$\frac{1}{\sqrt{\frac{L}{L_0} + \frac{1}{3}}}$	Difference, per cent
0.0	1.571	1.732	10.3	3.2	0.532	0.532	0.1
.1	1.429	1.519	6.3	3.3	.524	.525	.1
.2	1.314	1.369	4.2	3.4	.517	.518	.1
.3	1.220	1.257	3.0	3.5	.510	.511	.1
.4	1.142	1.168	2.3	3.6	.504	.504	.0
.5	1.077	1.095	1.7	3.7	.4977	.4979	.0
.6	1.021	1.035	1.4	3.8	.4916	.4919	.0
.7	.973	.984	1.1	3.9	.4859	.4860	.0
.8	.931	.939	.9	4.0	.4801	.4804	.0
.9	.894	.900	.7	4.5	.4548	.4549	.0
1.0	.860	.866	.7	5.0	.4330	.4330	.0
1.1	.831	.835	.5	5.5	.4141
1.2	.804	.808	.5	6.0	.3974
1.3	.779	.782	.4	6.5	.3826
1.4	.757	.760	.4	7.0	.3693
1.5	.736	.739	.4	7.5	.3574
1.6	.717	.719	.3	8.0	.3465
1.7	.699	.701	.3	8.5	.3366
1.8	.683	.685	.3	9.0	.3275
1.9	.668	.669	.3	9.5	.3189
2.0	.653	.655	.3	10.0	.3111
2.1	.640	.641	.2	11.0	.2972
2.2	.627	.628	.2	12.0	.2850
2.3	.615	.616	.2	13.0	.2741
2.4	.604	.605	.2	14.0	.2644
2.5	.593	.594	.2	15.0	.2556
2.6	.583	.584	.2	16.0	.2476
2.7	.574	.574	.2	17.0	.2402
2.8	.564	.565	.1	18.0	.2338
2.9	.556	.556	.1	19.0	.2277
3.0	.547	.548	.1	20.0	.2219
3.1	.539	.540	.1				

or

$$\frac{\cot \omega \sqrt{C_0 L_0}}{\omega \sqrt{C_0 L_0}} = \frac{L}{L_0} \quad (8)$$

This equation has been given by Guyau² and L. Cohen³. It determines the periodicity ω and hence the frequency and wave length of the possible natural modes of oscillation when the distributed capacity and inductance of the aerial and the inductance of the loading coil are known. This equation can not, however, be solved directly; it may be solved graphically, as shown in Fig. 4, or a table may be prepared indirectly which gives the values of $\omega \sqrt{C_0 L_0}$ for different values of $\frac{L}{L_0}$ from which then ω, f, λ may be determined. The second column of Table 1 gives these values for the lowest natural frequency of oscillation, which is of the major importance practically.

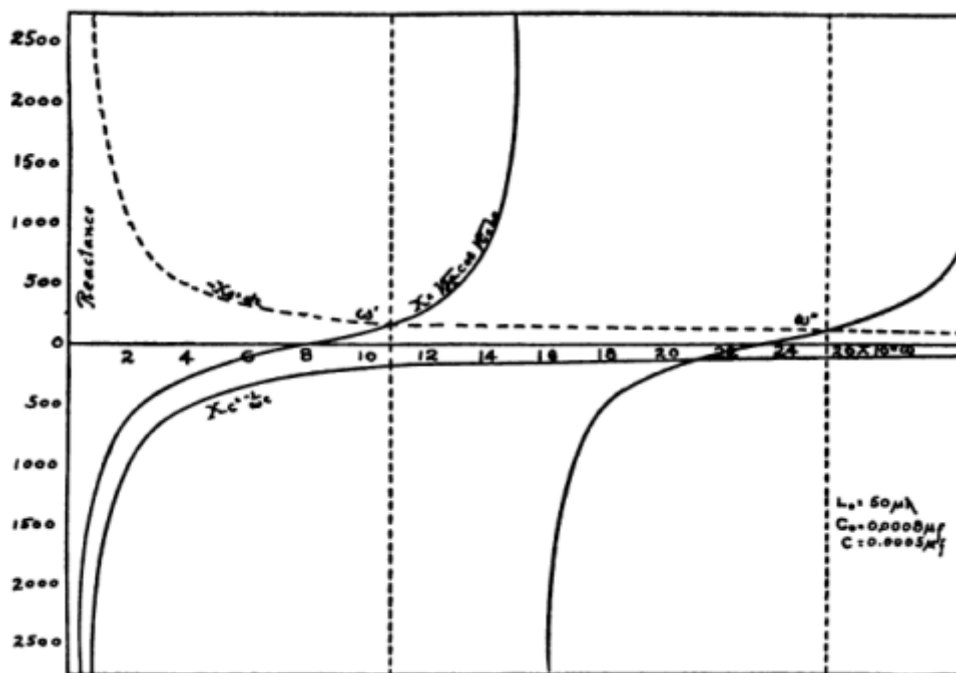


FIG. 5 - Curves of aerial and series condenser reactance

(b) *Condenser in Lead-in.* - At times in practice a condenser is inserted in the lead-in. If the capacity of the condenser is C , its reactance is $X_0 = -\frac{1}{\omega C}$. This reactance is shown in Fig. 5 by the hyperbola drawn in solid line. The intersection of the negative of this curve (drawn in dash line) with the cotangent curves representing X gives the frequencies for which $X_0 + X = 0$, and hence the natural frequencies of oscillation of the antenna. The frequencies are increased (the wave length decreased) by the insertion of the condenser and the oscillations of higher frequencies are not integral multiples of the lowest.

The condition $X_0 + X = 0$ is expressed by the equation

$$-\frac{\tan \omega \sqrt{C_0 L_0}}{\omega \sqrt{C_0 L_0}} = \frac{C}{C_0} \quad (9)$$

2 A. Guyau, *Lumiere Electrique*, 15, p. 13; 1911.

3 L. Cohen, *Electrical World*, 65, p. 286; 1915

which has also been given by Guyau. Equation (9) may be solved graphically as above or a table similar to Table 1 may be prepared giving $\omega\sqrt{C_0L_0}$ for different values of $\frac{C}{C_0}$. More complicated circuits may be solved in a similar manner.

3. EFFECTIVE RESISTANCE, INDUCTANCE, AND CAPACITY

In the following the most important practical case of a loading coil in the lead-in and the natural oscillation of lowest frequency alone will be considered. The problem is to replace the antenna

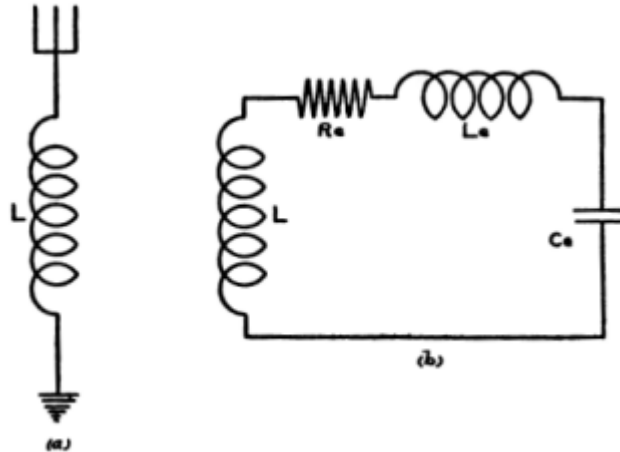


FIG. 6 - (a) Antenna with loading coil; (b) artificial antenna with lumped constant: to represent antenna in (a)

of Fig. 6, (a), which has a loading coil L in the lead-in and an aerial with distributed characteristics, by a circuit (Fig. 6, (b)) consisting of the inductance L in series with lumped resistance R_e , inductance L_e , capacity C_e , which are equivalent to the aerial. It is necessary, however, to state how these effective values are to be defined.

In practice the quantities which are of importance in an antenna are the resonant wave length or frequency and the current at the current maximum. The quantities L_e and C_e are therefore defined as those which will give the circuit (b) the same resonant frequency as the antenna in (a). Further the three quantities L_e , C_e , and R_e must be such that the current in (b) will be the same as the maximum in the antenna for the same applied emf whether undamped or damped with any decrement. These conditions determine L_e , C_e , and R_e uniquely at any given frequency and are the proper values for an artificial antenna which is to represent an actual antenna at a particular frequency. In the two circuits the corresponding maxima of magnetic energies and electrostatic energies and the dissipation of energy will be the same.

Zenneck⁴ has shown how these effective values of inductance capacity and resistance can be computed when the current and voltage distributions are known. Thus, if at any point x on the oscillator the current i and the voltage v are given by

$$i = I f(x); v = V \phi(x)$$

where I is the value of the current at the current loop and V the maximum voltage, then the differential equation of the oscillation is

⁴ Zenneck, Wireless Telegraphy (translated by A. E. Selig), Note 40, p. 430.

$$\frac{\delta I}{\delta t} \int R_1 f(x)^2 dx + \frac{\delta^2 I}{\delta t^2} \int L_1 f(x)^2 dx + \frac{I}{\int C_1 \phi(x)^2 dx} \left(\int C_1 \phi(x) dx \right)^2 = 0$$

where the integrals are taken over the whole oscillator. If we write

$$R_e = \int R_1 f(x)^2 dx \quad (10)$$

$$L_e = \int L_1 f(x)^2 dx \quad (11)$$

$$C_e = \frac{\left(\int C_1 \phi(x) dx \right)^2}{\int C_1 \phi(x)^2 dx} \quad (12)$$

the equation becomes

$$R_e \frac{\delta I}{\delta t} + L_e \frac{\delta^2 I}{\delta t^2} + \frac{I}{C_e} = 0$$

which is the differential equation of oscillation of a simple circuit with lumped resistance, inductance, and capacity of values R_e , L_e , and C_e and in which the current is the same as the maximum in the distributed case. In order to evaluate these quantities, it is necessary only to determine $f(x)$ and $\phi(x)$; that is, the functions which specify the distribution of current and voltage on the oscillator. In this connection it will be assumed that the resistance is not of importance in determining these distributions.

At the far end of the aerial the current is zero; that is, for $x = l$; $i_l = 0$. From equations (3) and (4) for $x = l$

$$v_l = \cos \omega t (A \cos \omega \sqrt{C_1 L_1} l + B \sin \omega \sqrt{C_1 L_1} l) \quad ,$$

$$i_l = \sin \omega t \sqrt{\frac{C_1}{L_1}} (A \sin \omega \sqrt{C_1 L_1} l - B \cos \omega \sqrt{C_1 L_1} l) \quad ;$$

and since $i_l = 0$,

$$A \sin \omega \sqrt{C_1 L_1} l = B \cos \omega \sqrt{C_1 L_1} l \quad .$$

From (3), then, we obtain

$$v = v_l \cos(\omega \sqrt{C_1 L_1} l - \omega \sqrt{C_1 L_1} x) \quad .$$

Hence

$$\phi(x) = \cos(\omega \sqrt{C_1 L_1} l - \omega \sqrt{C_1 L_1} x) \quad .$$

Now for $x = 0$ from (4) we obtain

$$i_0 = -B \sqrt{\frac{C_1}{L_1}} \sin \omega t = -A \sqrt{\frac{C_1}{L_1}} \tan \omega \sqrt{C_1 L_1} l \sin \omega t ,$$

whence

$$i = i_0 \frac{\sin(\omega \sqrt{C_1 L_1} l - \omega \sqrt{C_1 L_1} x)}{\sin \omega \sqrt{C_1 L_1} l}$$

and

$$f(x) = \frac{\sin(\omega \sqrt{C_1 L_1} l - \omega \sqrt{C_1 L_1} x)}{\sin \omega \sqrt{C_1 L_1} l} .$$

We can now evaluate the expressions (10), (11), and (12). From (10)

$$\begin{aligned} R_e &= \int_0^l R_1 \frac{\sin^3(\omega \sqrt{C_1 L_1} l - \omega \sqrt{C_1 L_1} x) dx}{\sin^2 \omega \sqrt{C_1 L_1} l} \\ &= \frac{R_1}{\sin^2 \omega \sqrt{C_1 L_1} l} \left[\frac{l}{2} - \frac{\sin 2 \omega \sqrt{C_1 L_1} l}{4 \omega \sqrt{C_1 L_1}} \right] \\ &= \frac{R_0}{2} \left[\frac{1}{\sin^2 \omega \sqrt{C_0 L_0}} - \frac{\cot \omega \sqrt{C_0 L_0}}{\omega \sqrt{C_0 L_0}} \right] \quad (13) \end{aligned}$$

and from (11) which contains the same form of integral

$$L_e = \frac{L_0}{2} \left[\frac{1}{\sin^2 \omega \sqrt{C_0 L_0}} - \frac{\cot \omega \sqrt{C_0 L_0}}{\omega \sqrt{C_0 L_0}} \right] \quad (14)$$

and from (12)

$$\begin{aligned} C_e &= \frac{(\int_0^l C_1 \cos(\omega \sqrt{C_1 L_1} l - \omega \sqrt{C_1 L_1} x) dx)^2}{\int_0^l C_1 \cos^2(\omega \sqrt{C_1 L_1} l - \omega \sqrt{C_1 L_1} x) dx} \\ &= \frac{C_1^2 \frac{\sin^2 \omega \sqrt{C_1 L_1} l}{(\omega \sqrt{C_1 L_1})^2}}{C_1 \left(\frac{l}{2} + \frac{\sin 2 \omega \sqrt{C_1 L_1} l}{4 \omega \sqrt{C_1 L_1}} \right)} \\ &= C_0 \frac{1}{\left[\frac{\omega \sqrt{C_0 L_0} \cot \omega \sqrt{C_0 L_0}}{2} + \frac{\omega^2 C_0 L_0}{2 \sin^2 \omega \sqrt{C_0 L_0}} \right]} \quad (15) \end{aligned}$$

The expressions (14) and (15) should lead to the same value for the reactance X of the aerial as obtained before. It is readily shown that

$$X = \omega L_e - \frac{1}{\omega C_e} = -\sqrt{\frac{L_0}{C_0}} \cot \omega \sqrt{C_0 L_0}$$

agreeing with equation (6).

It is of interest to investigate the values of these quantities at very low frequencies ($\omega = 0$), frequently called the static values, and those corresponding to the natural frequency of the unloaded antenna or the so-called fundamental of the antenna. Substituting $\omega = 0$ in (13), (14), and (15), and evaluating the indeterminate which enters in the first two cases, we obtain for the low-frequency values

$$\begin{aligned} R_e &= \frac{R_0}{3} \\ L_e &= \frac{L_0}{3} \quad (16) \\ C_e &= C_0 \end{aligned}$$

At low frequencies the current is a maximum at the lead-in end of the aerial and falls off linearly to zero at the far end. The effective resistance and inductance are one-third of the values which would obtain if the current were the same throughout. The voltage is, however, the same at all points and hence the effective capacity is the capacity per unit length times the length, or C_0 .

At the fundamental of the antenna, the reactance X of equation (6) becomes equal to zero and hence $\omega \sqrt{C_0 L_0} = \frac{\pi}{2}$. Substituting this value in (13), (14), and (15),

$$\begin{aligned} R_e &= \frac{R_0}{2} \\ L_e &= \frac{L_0}{2} \quad (17) \\ C_e &= \frac{8}{\pi^2} C_0 \end{aligned}$$

Hence, in going from low frequencies up to that of the fundamental of the antenna, the resistance (neglecting radiation and skin effect) and the inductance (neglecting skin effect) increase by 50 per cent, the capacity, however, decreases by about 20 per cent. The incorrect values $\frac{2}{\pi} L_0$ and $\frac{2}{\pi} C_0$ have been frequently given and commonly used as the values of the effective inductance and capacity of the antenna at its fundamental. These lead also to the incorrect value $L_e = \frac{L_0}{2}$ for the low-frequency inductance.⁵

⁵ These values are given by J.H. Morecroft in *Proc. I.R.E.*, 5, p. 389; 1917. It may be shown that they lead to correct values for the reactance of the aerial and hence to correct values of frequency, as was verified by the experiments. They are not, however, the values which would be correct for an artificial antenna in which the current must be equal to the maximum in the actual antenna and in which the energies must also be equal to those in the antenna. The resistance values given by Prof. Morecroft agree with these requirements and with the values obtained here.

The values for other frequencies may be obtained by substitution in (13), (14), (15). If the value L of the loading coil in the lead-in is given, the quantity $\omega\sqrt{C_0 L_0}$ is directly obtained from Table 1.

4. EQUIVALENT CIRCUIT WITH LUMPED CONSTANTS

In so far as the frequency or wave length is concerned, the aerial of the antenna may be considered to have constant values of inductance and capacity and the values of frequency or wave length for different loading coils can be computed with slight error using the simple formula applicable to circuits with lumped inductance and capacity. The values of inductance and capacity ascribed to the aerial are the static or low frequency; that is, $\frac{L_0}{3}$ for the inductance and C_0 , for the capacity. The total inductance in case the loading coil has a value L will be $L + \frac{L_0}{3}$ and the frequency is given by

$$f = \frac{1}{2\pi\sqrt{(L + \frac{L_0}{3})C_0}} \quad (18)$$

or the wave length in meters by

$$\lambda = 1884\sqrt{(L + \frac{L_0}{3})C_0} \quad (19)$$

where the inductance is expressed in microhenrys and the capacity in microfarads. The accuracy with which this formula gives the wave length can be determined by comparison with the exact formula (8). In the second column of Table 1 are given the values of $\omega\sqrt{C_0 L_0}$ for different values of $\frac{L}{L_0}$ as computed by formula (8). Formula (18) may be written in the form

$$\omega\sqrt{C_0 L_0} = \frac{1}{\sqrt{\frac{L}{L_0} + \frac{1}{3}}} \quad \text{so that the values of } \omega\sqrt{C_0 L_0} \text{ which are proportional to the frequency,}$$

may readily be computed from this formula also. These values are given in the third column and the per cent differences in the fourth column of Table 1. It is seen that formula (18) gives values for the frequency which are correct to less than 1 per cent excepting when very close to the fundamental of the antenna; that is, for very small values of L . Under these conditions the simple formula leads to values of the frequency which are too high. Hence to the degree of accuracy shown, which is amply sufficient in most practical cases, the aerial can be represented by its static inductance $\frac{L_0}{3}$ with its static capacity C_0 , in series, and the frequency of oscillation with a loading coil L in the lead-in can be computed by the ordinary formula applicable to circuits with lumped constants.

In an article by L. Cohen,⁶ which has been copied in several other publications, it was stated that the use of the simple wave length formula would lead to very large errors when applied to the antenna with distributed constants. The large errors found by Cohen are due to his having used the value L_0 , for the inductance of the aerial, instead of $\frac{L_0}{3}$, in applying the simple formula.

6 See footnote 3.

5. DETERMINATION OF STATIC CAPACITY AND INDUCTANCE

In applying formula (8) to calculate the frequency of a loaded antenna, a knowledge of the quantities of L_0 , and C_0 is required. In applying formula (18), $\frac{L_0}{3}$ and C_0 , are required. Hence either formula. may be used if the static capacity and inductance values are known. We will call these values simply the capacity C_a and inductance L_a of the antenna. Hence $C_a = C_0$, $L_a = \frac{L_0}{3}$ and the wave length from (19) is given by

$$\lambda = 1884 \sqrt{(L + L_a) C_a} \quad (20)$$

where inductance is expressed in microhenrys and capacity in microfarads, as before.

The capacity and inductance of the antenna are then readily determined experimentally by the familiar method of inserting, one after the other, two loading coils of known values L_1 and L_2 in the lead-in and determining the frequency of oscillation or wave length for each. From the observed wave lengths λ_1 and λ_2 and known values of the inserted inductances, the inductance of the antenna is given by

$$L_a = \frac{L_1 \lambda_2^2 - L_2 \lambda_1^2}{\lambda_1^2 - \lambda_2^2} \quad (21)$$

and the capacity of the antenna from either

$$\begin{aligned} \lambda_1 &= 1884 \sqrt{(L_1 + L_a) C_a} \\ \lambda_2 &= 1884 \sqrt{(L_2 + L_a) C_a} \end{aligned} \quad (22)$$

using, preferably, the equation corresponding to the larger valued coil. This assumes that formula (20) holds exactly.

As an example, let us assume that the antenna has $L_0 = 50$ microhenrys and $C_0 = 0.001$ microfarad and that we insert two coils of 50 and 150 microhenrys and determine the wave lengths, experimentally. We know from formula (8) and Table 1 that the wave lengths would be found to be 491 and 771 m. From the observed wave lengths and known inductances, the value of L_a would be found by (21) to be

$$L_a = 17.8 \text{ microhenrys}$$

and from (22)

$$C_a = 0.00099, \text{ microfarad.}$$

C_a is very close to the assumed value C_0 but L_a differs by 7 per cent from $\frac{L_0}{3}$. This accuracy would ordinarily be sufficient. We can, however, by a second approximation, derive from the experimental data a more accurate value of L_a . For, the observed value of L_a furnishes rough values of $\frac{L_1}{L_0}$ and $\frac{L_2}{L_0}$, which in this example come out 0.96 and 2.88, respectively. But Table 1 gives

the per cent error of formula (20) for different values of $\frac{L}{L_0}$ and shows that this formula gives a 0.7 per cent shorter wave length than 491 m (or 488 m) for $\frac{L}{L_0}=0.96$ but no appreciable difference for $\frac{L}{L_0}=2.88$. Recomputing L_a , using 488 and 771 m, gives

$$L_a = 0.0168$$

which is practically identical with the assumed $\frac{L_0}{3}$.

6. DETERMINATION OF EFFECTIVE RESISTANCE, INDUCTANCE, AND CAPACITY

When a source of undamped oscillations in a primary circuit induces current in a secondary tuned circuit, the current in the secondary, for a given emf, depends only upon the resistance of the secondary circuit. When damped oscillations are supplied by the source in the primary, the current in the secondary, for a given emf and primary decrement, depends upon the decrement of the secondary; that is, upon the resistance and ratio of capacity to inductance. The higher the decrement of the primary circuit relative to the decrement of the secondary the more strongly does the current in the secondary depend upon its own decrement. This is evident from the expression for the current I in the secondary circuit.

$$P = \frac{N E_0^2}{4f R^2 \delta' \left(1 + \frac{\delta'}{\delta}\right)}$$

where δ' is the decrement of the primary, δ that of the secondary, R the resistance of the secondary, f the frequency, E_0 the maximum value of the emf impressed on the primary, and N the wave-train frequency.

These facts suggest a method of determining the effective resistance, inductance, and capacity of an antenna at a given frequency in which all of the measurements are made at one frequency and which does not require any alteration of the antenna circuit whatsoever. The experimental circuits are arranged as shown in Fig. 7, where S represents a coil in the primary circuit which may be thrown either into the circuit of a source of undamped or of damped oscillations. The coil L is the loading coil of the antenna, which may be thrown over to the measuring circuit containing a variable inductance L' , a variable condenser C' , and variable resistance R' . The condenser C' should be resistance free and shielded, the shielded terminal being connected to the ground side. First, the undamped source is tuned to the antenna, and then the $L'C'$ circuit tuned to the source. The resistance R' is then varied until the current is the same in the two positions. The resistance of the $L'C'$ circuit is then equal to R_e , the effective resistance of the aerial-ground portion of the antenna and $L'C' = L_e C_e$. Next, the damped source is tuned to the antenna and the change in current noted when the connection is thrown over to the $L'C'$ circuit. If the current increases, the value of C' is greater than C_e , and vice versa. By varying both L' and C' ,

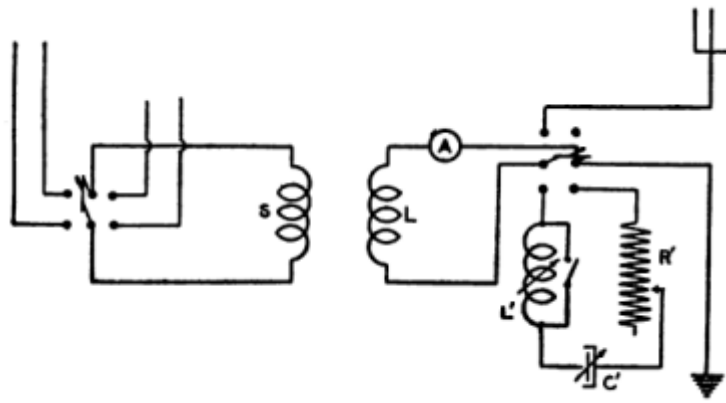


FIG. 7 - Circuits for determining the effective resistance, inductance, and capacity of an antenna

keeping the tuning and R' unchanged, the current can be adjusted to the same value in both positions. Then, since $L'C' = L_e C_e$ and $\frac{C'}{L'} = \frac{C_e}{L_e}$, the value of C' gives C_e and that of L' gives L_e .

Large changes in the variometer setting may result in appreciable changes in its resistance so that the measurement should be repeated after the approximate values have been found. To eliminate the resistance of the variometer in determining R_e , the variometer is shorted and, using undamped oscillations, the resonance current is adjusted to equality in the two positions by varying R' . Then $R' = R_e$. The measurement requires steady sources of feebly damped and strongly damped current. The former is readily obtained by using a vacuum-tube generator. A resonance transformer and magnesium spark gap operating at a low-spark frequency serve very satisfactorily for the latter source, or a single source of which the damping can be varied will suffice. An accuracy of 1 per cent is not difficult to obtain.

IV. THE INDUCTANCE COIL

The transmission-line theory can also be applied to the treatment of the effects of distributed capacity in inductance coils. In Fig. 8, (a), is represented a single-layer solenoid connected to a variable condenser C . A and B are the terminals of the coil, D the middle, and the condensers drawn in dotted lines are supposed to represent the capacities between the different parts of the coil. In Fig. 8, (b), the same coil is represented as a line with uniformly distributed inductance and capacity.

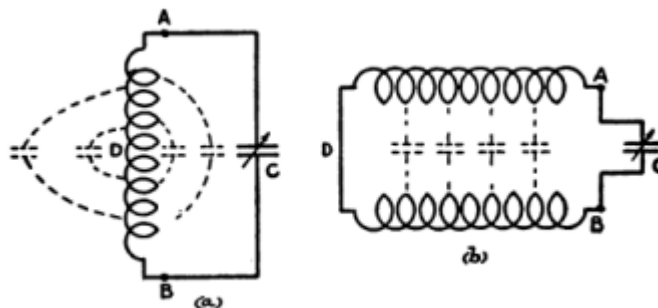


FIG. 8 - Inductance coil represented as a line with uniform distribution of inductance and capacity

These assumptions are admittedly rough but are somewhat justified by the known similarity of the oscillations in long solenoids to those in a simple antenna.

1. REACTANCE OF THE COIL

Using the same notation as before, an expression for the reactance of the coil, regarded from the terminals AB ($x = 0$) will be determined considering the line as closed at the far end D ($x = l$). Equations (3) and (4) will again be applied, taking account of the new terminal condition: that is, for $x = l$; $v = 0$. Hence

$$A \cos \omega \sqrt{C_1 L_1} l = -B \sin \omega \sqrt{C_1 L_1} l$$

and for $x = 0$

$$v_0 = A \cos \omega t = -B \tan \omega \sqrt{C_1 L_1} \cos \omega t$$

$$i_0 = -\sqrt{\frac{C_1}{L_1}} B \sin \omega t$$

which gives for the reactance of the coil regarded from the terminals AB ,

$$X' = \sqrt{\frac{L_1}{C_1}} \tan \omega \sqrt{C_1 L_1} l$$

or

$$X' = \sqrt{\frac{L_0}{C_0}} \tan \omega \sqrt{C_0 L_0} \quad (23)$$

2. NATURAL FREQUENCIES OF OSCILLATION

At low frequencies the reactance of the coil is very small and positive but increases with increasing frequency and becomes infinite when $\omega \sqrt{C_0 L_0} = \frac{\pi}{2}$. This represents the lowest frequency of natural oscillation of the coil when the terminals are open. Above this frequency the reactance is highly negative, approaching zero at the frequency $\omega \sqrt{C_0 L_0} = \pi$. In this range of frequencies the coil behaves as a condenser and would require an inductance across the terminals to form a resonant circuit. At the frequency $\omega \sqrt{C_0 L_0} = \pi$ the coil will oscillate with its terminals short-circuited. As the frequency is still further increased the reactance again becomes increasingly positive.

Condenser Across the Terminals. - The natural frequencies of oscillation of the coil when connected to a condenser C are given by the condition that the total reactance of the circuit shall be zero.

$$X' + X_0 = 0$$

From this we have

$$\sqrt{\frac{L_0}{C_0}} \tan \omega \sqrt{C_0 L_0} = \frac{1}{\omega C}$$

or

$$\frac{\cot \omega \sqrt{C_0 L_0}}{\omega \sqrt{C_0 L_0}} = \frac{C}{C_0} \quad (24)$$

This expression is the same as (8) obtained in the case of the loaded antenna, excepting that $\frac{C}{C_0}$ occurs on the right-hand side instead of $\frac{L}{L_0}$ and shows that the frequency is decreased and wave length increased by increasing the capacity across the coil in a manner entirely similar to the decrease in frequency produced by inserting loading coils in the antenna lead-in.

3. EQUIVALENT CIRCUIT WITH LUMPED CONSTANTS

It is of interest to investigate the effective values of inductance and capacity of the coil at very low frequencies.

Expanding the tangent in equation (23) into a series we find

$$X' = \omega L_0 \left(1 + \frac{\omega^2 C_0 L_0}{3} + \dots \right)$$

and neglecting higher-power terms this may be written

$$X' = \frac{(\omega L_0) \left(-\frac{3}{\omega C_0} \right)}{\omega L_0 - \frac{3}{\omega C_0}}$$

This is the reactance of an inductance L_0 in parallel with a capacity $\frac{C_0}{3}$, which shows that at low frequencies the coil may be regarded as an inductance L_0 , with a capacity $\frac{C_0}{3}$ across the terminals and therefore in parallel with the external condenser C . Since at low frequencies the current is uniform throughout the whole coil, it is self-evident that its inductance should be L_0 .

Now, the similarity between equations (24) and (8) shows that, just as accurately as in the similar case of the loaded antenna, the frequency of oscillation of a coil with *any capacity* C across the terminals is given by the formula

$$f = \frac{1}{2\pi \sqrt{L_0 \left(C + \frac{C_0}{3} \right)}}$$

This, however, is *also* the expression for the frequency of a coil of pure inductance L_0 with a capacity $\frac{C_0}{3}$ across its terminals and which is in parallel with an external capacity C . Therefore, in so far as frequency relations are concerned, an inductance coil with distributed capacity is closely equivalent at any frequency to a pure inductance, equal to the low-frequency inductance (neglecting

skin effect), with a constant capacity across its terminals. This is a well-known result of experiment⁷, at least in the case of single-layer solenoids which, considering the changes in current and voltage distribution in the coil with changing frequency, is not otherwise self-evident.

WASHINGTON, March 9, 1918.

⁷ G.W.O. Howe, Proc. Phys. Soc. London, 21 p. 251, 1912; F.A. Kolster, Proc. Inst. Radio Eng., 1, p. 19, 1913; J.C. Hubbard, Phys. Rev., 9, p. 529, 1917.

FURTHER DISCUSSION* ON
"ELECTRICAL OSCILLATIONS IN ANTENNAS AND
INDUCTION COILS" BY JOHN M. MILLER

By
John H. Morecroft

I was glad to see an article by Dr. Miller on the subject of oscillations in coils and antennas because of my own interest in the subject, and also because of the able manner in which Dr. Miller handles material of this kind. The paper is well worth studying.

I was somewhat startled, however, to find out from the author that I was in error in some of the material presented in my paper in the Proceedings of The Institute of Radio Engineers for December, 1917, especially as I had at the time I wrote my paper thought along similar lines as does Dr. Miller in his treatment of the subject: this is shown by my treatment of the antenna resistance.

As to what the effective inductance and capacity of an antenna are when it is oscillating in its fundamental mode is, it seems to me, a matter of viewpoint. Dr. Miller concedes that my treatment leads to correct predictions of the behavior of the antenna and I concede the same to him; it is a question, therefore, as to which treatment is the more logical.

From the author's deductions we must conclude that at quarter wave length oscillations

$$L_e = \frac{L_0}{2} \quad (1)$$

and

$$C_e = \frac{8}{\pi^2} C_0 \quad (2)$$

The value of L really comes from a consideration of the magnetic energy in the antenna keeping the current in the artificial antenna the same as the maximum value it had in the actual antenna, and then selecting the capacity of suitable value to give the artificial antenna the same natural period as the actual antenna. This method of procedure will, as the author states, give an artificial antenna having the same natural frequency, magnetic energy, and electrostatic energy, as the actual antenna, keeping the current in the artificial antenna the same as the maximum current in the actual antenna.

But suppose he had attacked the problem from the viewpoint of electrostatic energy instead of electromagnetic energy, and that he had obtained the constants of his artificial antenna to satisfy these conditions (which are just as fundamental and reasonable as those he did satisfy); same natural frequency, same magnetic energy, same electrostatic energy and the same voltage across the condenser of the artificial as the maximum voltage in the actual antenna. He would then have obtained the relations

$$L_e = \frac{8}{\pi^2} L_0 \quad (3)$$

and

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$$C_e = \frac{C_0}{2} \quad (4)$$

Now equations (3) and (4) are just as correct as are (1) and (2) and moreover the artificial antenna built with the constants given in (3) and (4) would duplicate the actual antenna just as well as the one built according to the relations given in (1) and (2).

I had these two possibilities in mind when writing in my original article "as the electrostatic energy is a function of the potential curve and the magnetic energy is the same function of the current curve, and both these curves have the same shape, it is logical, *and so on.*" Needless to say, I still consider it logical, and after reading this discussion I am sure Dr. Miller will see my reasons for so thinking.

When applying the theory of uniform lines to coils I think a very large error is made at once, which vitiates very largely any conclusions reached. The L and C of the coil, per centimeter length, are by no means uniform, a necessary condition in the theory of uniform lines; in a long solenoid the L per centimeter near the center of the coil is nearly twice as great as the L per centimeter at the ends, a fact which follows from elementary theory, and one which has been verified in our laboratory by measuring the wave length of a high frequency wave traveling along such a solenoid. The wave length is much shorter in the center of the coil than it is near the ends. What the capacity per centimeter of a solenoid is has never been measured, I think, but it is undoubtedly greater in the center of the coil than near the ends.

The conclusions he reaches from his equation (22) that even at its natural frequency the L of the coil may be regarded as equal to the low frequency value of L is valuable in so far as it enables one better to predict the behavior of the coil, but it should be kept in mind that really the value of L of the coil, when defined as does the author in the first part of his paper in terms of magnetic energy and maximum current in the coil at the high frequency, is very much less than it is at the low frequency.

One point on which I differ very materially with the author is the question of the reactance of a coil and condenser, connected in parallel, and excited by a frequency the same as the natural frequency of the circuit. The author gives the reactance as infinity at this frequency, whereas it is actually zero. When the impressed frequency is slightly higher than resonant frequency there is a high capacitive reaction and at a frequency slightly lower than resonant frequency there is a high inductive reaction, but at the resonant frequency the reactance of the circuit is zero. The resistance of the circuit becomes infinite at this frequency, if the coil and condenser have no resistance, but for any value of coil resistance, the reactance of the combination is zero at resonant frequency.